Transients Solutions by Laplace Transform

* Laplace Transforms:

	$f(s)$	$F(t)$
1	$\frac{1}{s}$	1
2	$\frac{1}{s^2}$	t
3	$\frac{1}{s^n} \quad (n = 1, 2, \ldots)$	$\frac{t^{n-1}}{(n-1)!}$
4	$\frac{1}{s+\alpha}$	$\epsilon^{-\alpha t}$
5	$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-\epsilon^{-\alpha t})$
6	$\frac{1}{(s+\alpha)^2}$	$t\epsilon^{-\alpha t}$
7	$\frac{1}{(s+\alpha)^n} \quad (n = 1, 2, \ldots)$	$\frac{t^{n-1}\epsilon^{-\alpha t}}{(n-1)!}$
8	$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\epsilon^{-\alpha t} + \alpha t - 1)$
9	$\frac{1}{s^3(s+\alpha)}$	$\frac{1}{\alpha^2}\left(\frac{1}{\alpha} - t + \frac{\alpha t^2}{2} - \frac{1}{\alpha}\epsilon^{-\alpha t}\right)$
10	$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{(\beta-\alpha)}(\epsilon^{-\alpha t} - \epsilon^{-\beta t})$
11	$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{(\beta-\alpha)}(\beta\epsilon^{-\beta t} - \alpha\epsilon^{-\alpha t})$
12	$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1 - \epsilon^{-\alpha t} - \alpha t\epsilon^{-\alpha t})$
13	$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^3}(\alpha t - 2 + \alpha t\epsilon^{-\alpha t} + 2\epsilon^{-\alpha t})$
14	$\frac{1}{(s+\alpha)(s+\beta)^2}$	$\frac{1}{(\alpha-\beta)^2}\{\epsilon^{-\alpha t} + [(\alpha-\beta)t - 1]\epsilon^{-\beta t}\}$
15	$\frac{1}{(s+\alpha)(s+\beta)(s+\gamma)}$	$\frac{1}{(\beta-\alpha)(\gamma-\alpha)}\epsilon^{-\alpha t} + \frac{1}{(\alpha-\beta)(\gamma-\beta)}\epsilon^{-\beta t} + \frac{1}{(\alpha-\gamma)(\beta-\gamma)}\epsilon^{-\gamma t}$
16	$\frac{1}{s^2+\alpha^2}$	$\frac{1}{\alpha}\sin \alpha t$
17	$\frac{s}{s^2+\alpha^2}$	$\cos \alpha t$
18	$\frac{1}{s(s^2+\alpha^2)}$	$\frac{1}{\alpha^2}(1 - \cos \alpha t)$
19	$\frac{1}{s^2(s^2+\alpha^2)}$	$\frac{1}{\alpha^3}(\alpha t - \sin \alpha t)$
20	$\frac{1}{(s^2+\alpha^2)^2}$	$\frac{1}{2\alpha^3}(\sin \alpha t - at \cos \alpha t)$
21	$\frac{s^2}{(s^2+\alpha^2)^2}$	$\frac{1}{2\alpha}(\sin \alpha t + \alpha t \cos \alpha t)$
22	$\frac{s^2-\alpha^2}{(s^2+\alpha^2)^2}$	$t \cos \alpha t$
23	$\frac{1}{s^2-\alpha^2}$	$\frac{1}{\alpha}\sinh \alpha t$
24	$\frac{s}{s^2-\alpha^2}$	$\cosh \alpha t$

* Circuits Representain using Laplace:

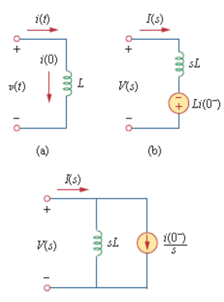


Figure 16.1
Representation of an inductor: (a) time-domain, (b,c) s-domain equivalents.

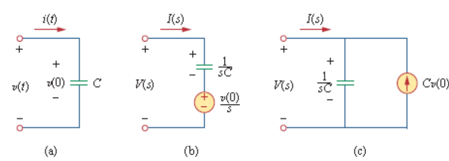


Figure 16.2
Representation of a capacitor: (a) time-domain, (b,c) s-domain equivalents.

Impedance of an element in the s-domain.*

Element	$Z(s) = V(s)/I(s)$
Resistor	R
Inductor	sL
Capacitor	$1/sC$

* Assuming zero initial conditions

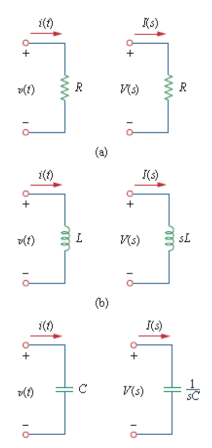
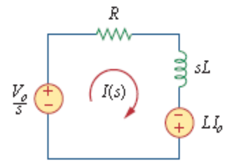


Figure 16.3
Time-domain and s-domain representations of passive elements under zero initial conditions.

Ex. RL Circuit:

Using KVL:

$$I(s)(R + sL) - LI_o - \frac{V_o}{s} = 0$$



or

$$I(s) = \frac{LI_o}{R + sL} + \frac{V_o}{s(R + sL)} = \frac{I_o}{s + R/L} + \frac{V_o/L}{s(s + R/L)}$$

using partial fraction and rearranging the equation:

$$I(s) = \frac{I_o}{s + R/L} + \frac{V_o/R}{s} - \frac{V_o/R}{(s + R/L)}$$

The inverse Laplace transform of this gives

$$i(t) = \left(I_o - \frac{V_o}{R}\right)e^{-t/\tau} + \frac{V_o}{R}, \qquad t \geq 0$$

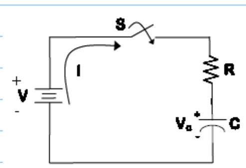
If $I_o = 0$,

$$i(t) = \frac{V_o}{R}(1 - e^{-t/\tau}), \qquad t \geq 0 \quad , \ \tau = \frac{L}{R}$$

* Ex. RC Circuits:

KCL:

$$\frac{V - V_c}{R} = C\frac{dV_c}{dt} \quad \text{or} \quad RC\frac{dV_c}{dt} + V_c = V$$



Rearranging:

$$V = V_c + RC\frac{dV_c}{dt}$$

Using Laplace table:

$$\frac{V}{s} = V_c(s) + RC(sV_c(s) - V_c(0)) \longrightarrow V_c(s) = \frac{V}{s} - \frac{V - V_c(0)}{\left(s + \frac{1}{RC}\right)} \longrightarrow V_c = V - [V - V_c(0)]e^{-\frac{t}{RC}}$$

Ex. LC Circuits:

Read section 2.3